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Estimation and Approximation of the Value of the Number "e"

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Abstract

The topic namely estimation and approximation of the values of "e" play a vital role at higher levels of education in the field of Mathematics, in other words this estimation method provides a strong foothold for acquiring higher knowledge of mathematics. This approximation is completely focused on the issue how a student of mathematics can be inculcated to understand the techniques of estimating errors.

Keywords: Estimation, Approximation, Knowledge, Infinite, Series.

Introduction

Mathematical programming applications often require an objective function to be approximated by one of simpler form so that an available computational approach can be used. An a priori bound is derived on the amount of error (suitably defined) which such an approximation can induce. This leads to a natural criterion for selecting the "best" approximation from any given class. We show that this criterion is equivalent for all practical purposes. Most applications of mathematical programming require the modeler to exercise some discretion in estimating or approximating the objective function to be optimized. We give a simple a priori bound relating the amount of objective function approximation error to the amount of error thereby induced in the solution of the corresponding optimization problem. This furnishes a natural criterion to guide the choice of an estimated or approximate objective function.

In this article we want to show how the error in the value of ''e'' can be minized by selecting a series in the infinite series of ''e'' The number ''e'' denotes the sum of the infinite series

$$e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+----------- +\frac{1}{n!}+------$$

$$Thus \quad e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+------+\frac{1}{n!}+--=\sum_{n=1}^{\infty}\frac{1}{n}$$

$$=\sum_{n=1(}^{\infty}\frac{1}{n-1)!} = ---- \sum_{n=r}^{\infty}\frac{1}{(n-r)!}$$

(n-r)! makes sense only n≥r.

The value of e lies between 2 and 3

$$We\ have \quad e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+--$$

$$1+1+\left(\frac{1}{2!}+\frac{1}{3!}+----\right) = 2+(a\ positive\ number\ > 2$$

$$\therefore e > 2 ------------------ (1)$$

$$Also\ e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+---- = 1+\frac{1}{1!}+\frac{1}{1.2}+\frac{1}{1.2.3}+--$$

$$= 1+1+\frac{1}{2}+\frac{1}{2.3}+\frac{1}{2.3.4}+---$$

$$< 1+1+\frac{1}{2}+\frac{1}{2.2}+\frac{1}{2.2.2}+---(because\ \frac{1}{3},\frac{1}{4}---\ are\ all < \frac{1}{2})$$

$$= 1+\ (1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+----)$$

$$= 1\ +\ (sum\ of\ an\ infinite\ G.P\ with\ a = 1, r = \frac{1}{2})$$

$$= 1+\ a/(1-r\ ) = \ 1+1/(1-1/2) = 1+2 = 3$$

$$\rightarrow e < 3 ----- (2)$$

$$Combining\ (1)\ and\ (2)\ we\ have\ 2 < e < 3$$

$$Also\ e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + --------$$

$$> 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} = 1 + \frac{1}{2} + \frac{1}{6} = \frac{16}{6} = \frac{8}{3} = 2.66 > 2.6$$

$$e > 2.6 \ \therefore \ 2.6 < e < 3$$

$$Also\ e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + --------$$

$$= (1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}) + \frac{1}{120} + ----$$

$$> 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = \frac{65}{24} = 2.708 > 2.7$$

$$e > 2.7 \quad \gg 2.7 < e < 3$$

$$Again\ e = (1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!}) + (\frac{1}{4!} + \frac{1}{5!} + ----)$$

$$= (1 + 1 + 0.5 + 0.166) + \frac{1}{24}(1 + \frac{1}{5} + \frac{1}{5.6} + ---)$$

$$= 2.666 + \frac{1}{24}(1 + \frac{1}{5} + \frac{1}{5.6} + ----)$$

$$< 2.666 + \frac{1}{24}\left(1 + \frac{1}{4} + \frac{1}{4^2} \pm ---\right) \ because$$

$$\frac{1}{5} < \frac{1}{4}; \frac{1}{5.6} < \frac{1}{4^2} ----$$

$$= 2.666 + \frac{1}{24}\left(\frac{1}{1 - \frac{1}{4}}\right) = 2.666 + \frac{1}{24} \times \frac{4}{3}$$

$$= 2.666 + \frac{1}{18}$$

$$= 2.666 + 0.056 = 2.722 \ \gg e < 2.722 \ \gg 2.7 < e, 2.722$$

$$\underline{now\ minimizing\ the\ error\ of\ e}$$

$$let\ E\ be\ the\ error\ than$$

$$E = \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + -----+$$

$$= \frac{1}{5!}(1 + \frac{1}{6} + \frac{1}{6.7} + \frac{1}{6.7.8} + ---)$$

$$< \frac{1}{120}(1 + \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + ---) = \frac{1}{120} \times \frac{6}{5} = \frac{1}{100}$$

$$E < \frac{1}{100}$$

$$by\ taking\ first\ five\ terms\ in\ the\ expansion\ of\ e\ the\ error\ committed\ is\ less\ than\ \frac{1}{100}$$

$$if\ E\ is\ the\ error\ than\ E = \frac{1}{6!} + \frac{1}{7!} + ---$$

$$= \frac{1}{5!}(\frac{1}{6} + \frac{1}{6.7} + \frac{1}{6.7.8} + ---)$$

$$< \frac{1}{5!}\left(\frac{1}{6} + \frac{1}{6^2} + ----\right)$$

$$\frac{1}{5!}\left(\frac{1/6}{1 - \frac{1}{6}}\right) = \frac{1}{5!} \times \frac{1}{6} \times \frac{6}{5} = \frac{1}{600}$$

$$thus\ E < \frac{1}{600}$$

$$again\ E = \frac{1}{6!} + \frac{1}{7!} + --= \frac{1}{6!}(1 + \frac{1}{7} + \frac{1}{7.8} + ---)$$

$$< \frac{1}{720}\left(1 + \frac{1}{7} + \frac{1}{7^2} + ----\right) = \frac{1}{720} \times \frac{1}{1 - \frac{1}{7}}$$

$$= \frac{1}{720} \times \frac{7}{6} = \frac{7}{4320} = 0.0016204 \approx 0.001$$

$$thus\ E < \frac{1}{1000}$$

By repeating the same process we come to the conclusion that we can minimize the error in the value of e and can refine its value to the extant we wish to.

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